UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 6TH FEBRUARY 2014

Organised by the **United Kingdom Mathematics Trust**
and supported by

Institute and Faculty of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

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1. What is 25 % of $\frac{3}{4}$?

A $\dfrac{3}{16}$ B $\dfrac{1}{4}$ C $\dfrac{1}{3}$ D 1 E 3

2. Which is the smallest positive integer for which all these are true?

(i) It is odd.

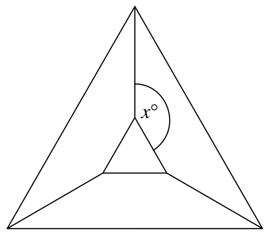
(ii) It is not prime.

(iii) The next largest odd integer is not prime.

A 9 B 15 C 21 D 25 E 33

3. An equilateral triangle is placed inside a larger equilateral triangle so that the diagram has three lines of symmetry.

What is the value of x?



A 100 B 110 C 120 D 130 E 150

4. You are given that m is an even integer and n is an odd integer. Which of these is an odd integer ?

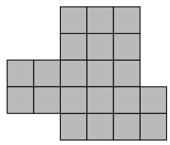
A $3m + 4n$ B $5mn$ C $(m + 3n)^2$ D m^3n^3 E $5m + 6n$

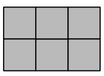
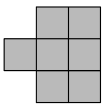
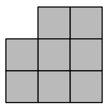
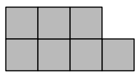
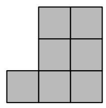
5. A ship's bell is struck every half hour, starting with one bell at 0030, two bells (meaning the bell is struck twice) at 0100, three bells at 0130 until the cycle is complete with eight bells at 0400. The cycle then starts again with one bell at 0430, two bells at 0500 and so on. What is the total number of times the bell is struck between 0015 on one day and 0015 on the following day?

A 24 B 48 C 108 D 144 E 216

6. The shape shown on the right was assembled from three identical copies of one of the smaller shapes below, without gaps or overlaps.

Which smaller shape was used?



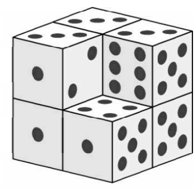
A  B  C  D  E 

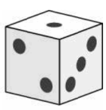
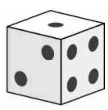
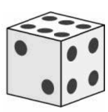
7. Just one positive integer has exactly 8 factors including 6 and 15.

What is the integer?

A 21 B 30 C 45 D 60 E 90

8. A large cube is made by stacking eight dice. The diagram shows the result, except that one of the dice is missing. Each die has faces with 1, 2, 3, 4, 5 and 6 pips and the total number of pips on each pair of opposite faces is 7. When two dice are placed face to face, the matching faces must have the same number of pips. What could the missing die look like?

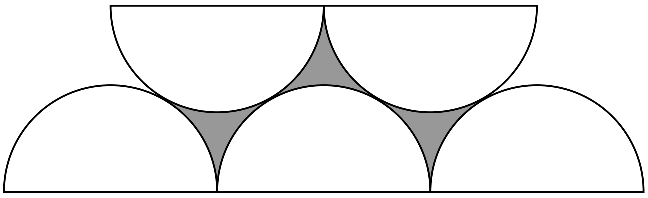


A  B  C  D  E 

9. At the age of twenty-six, Gill has passed her driving test and bought a car. Her car uses p litres of petrol per 100 km travelled. How many litres of petrol would be required for a journey of d km?

A $\frac{pd}{100}$ B $\frac{100p}{d}$ C $\frac{100d}{p}$ D $\frac{100}{pd}$ E $\frac{p}{100d}$

10. The diagram shows five touching semicircles, each with radius 2.



What is the length of the perimeter of the shaded shape?

A 5π B 6π C 7π D 8π E 9π

11. Not all characters in the Woodentops series tell the truth. When Mr Plod asked them, "How many people are there in the Woodentops family?", four of them replied as follows:

Jenny: "An even number." Willie: "An odd number." Sam: "A prime number."

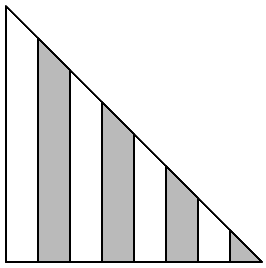
Mrs Scrubitt: "A number which is the product of two integers greater than one."

How many of these four were telling the truth?

A 0 B 1 C 2 D 3 E 4

12. The diagram shows an isosceles right-angled triangle divided into strips of equal width. Four of the strips are shaded.
What fraction of the area of the triangle is shaded?

A $\frac{11}{32}$ B $\frac{3}{8}$ C $\frac{13}{32}$ D $\frac{7}{16}$ E $\frac{15}{32}$



13. How many numbers can be written as a sum of two different positive integers each at most 100?

A 100 B 197 C 198 D 199 E 200

14. This year the *Tour de France* starts in Leeds on 5 July. Last year, the total length of the *Tour* was 3404 km and the winner, Chris Froome, took a total time of 83 hours 56 minutes 40 seconds to cover this distance. Which of these is closest to his average speed over the whole event?

A 32 km/h B 40 km/h C 48 km/h D 56 km/h E 64 km/h

15. Zac halves a certain number and then adds 8 to the result. He finds that he obtains the same answer if he doubles his original number and then subtracts 8 from the result.
What is Zac's original number?

A $8\frac{2}{3}$ B $9\frac{1}{3}$ C $9\frac{2}{3}$ D $10\frac{1}{3}$ E $10\frac{2}{3}$

16. The base of a triangle is increased by 25% but the area of the triangle is unchanged. By what percentage is the corresponding perpendicular height decreased?

A $12\frac{1}{2}\%$ B 16% C 20% D 25% E 50%

17. How many weeks are there in $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ minutes?

A 1 B 2 C 3 D 4 E 5

18. Consider looking from the origin $(0, 0)$ towards all the points (m, n), where each of m and n is an integer. Some points are *hidden*, because they are directly in line with another nearer point. For example, $(2, 2)$ is hidden by $(1, 1)$.

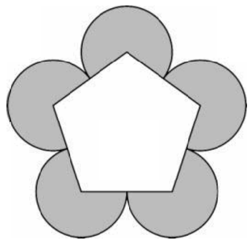
How many of the points $(6, 2)$, $(6, 3)$, $(6, 4)$, $(6, 5)$ are *not* hidden points?

A 0 B 1 C 2 D 3 E 4

19. Suppose that $8^m = 27$. What is the value of 4^m?

A 3 B 4 C 9 D 13.5 E there is no such m

20. The diagram shows a regular pentagon and five circular arcs. The sides of the pentagon have length 4. The centre of each arc is a vertex of the pentagon, and the ends of the arc are the midpoints of the two adjacent edges.



What is the total shaded area?

A 8π B 10π C 12π D 14π E 16π

21. In King Arthur's jousting tournament, each of the several competing knights receives 17 points for every bout he enters. The winner of each bout receives an extra 3 points. At the end of the tournament, the Black Knight has exactly one more point than the Red Knight.

What is the smallest number of bouts that the Black Knight could have entered?

A 3 B 4 C 5 D 6 E 7

22. The positive integers a, b and c are all different. None of them is a square but all the products ab, ac and bc are squares. What is the least value that $a + b + c$ can take?

A 14 B 28 C 42 D 56 E 70

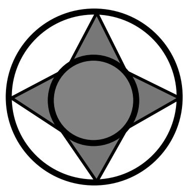
23. A sector of a disc is removed by making two straight cuts from the circumference to the centre. The perimeter of the sector has the same length as the circumference of the original disc. What fraction of the area of the disc is removed?

A $\frac{\pi - 1}{\pi}$ B $\frac{1}{\pi}$ C $\frac{\pi}{360}$ D $\frac{1}{3}$ E $\frac{1}{2}$

24. How many 4-digit integers (from 1000 to 9999) have at least one digit repeated?

A 62×72 B 52×72 C 52×82 D 42×82 E 42×92

25. The diagram shows two concentric circles with radii 1 and 2 units, together with a shaded octagon, all of whose sides are equal.



What is the length of the perimeter of the octagon?

A $8\sqrt{2}$ B $8\sqrt{3}$ C $8\sqrt{3}\pi$ D $2\sqrt{5 + 2\sqrt{2}}$ E $8\sqrt{5 - 2\sqrt{2}}$